



## Deontic Logic

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# M I N D

## A QUARTERLY REVIEW

OF

## PSYCHOLOGY AND PHILOSOPHY



### I. DEONTIC LOGIC

BY G. H. VON WRIGHT

1. So called modal concepts might conveniently be divided into three or four main groups. There are the alethic modes or modes of truth. These are concepts such as the necessary (the necessarily true), the possible (the possibly true), and the contingent (the contingently true). There are the epistemic modes or modes of knowing. These are concepts such as the verified (that which is known to be true), the undecided, and the falsified (that which is known to be false). There are the deontic<sup>1</sup> modes or modes of obligation. These are concepts such as the obligatory (that which we ought to do), the permitted (that which we are allowed to do), and the forbidden (that which we must not do). As a fourth main group of modal categories one might add the existential modes or modes of existence. These are concepts such as universality, existence, and emptiness (of properties or classes).

There are essential similarities but also characteristic differences between the various groups of modalities. They all deserve, therefore, a special treatment. The treatment of the existential modes is usually known as quantification theory. The treatment of the alethic modes covers most of what is traditionally known as modal logic. The epistemic modes have not to any great extent and the deontic modes hardly at all been treated by logicians.

In the present paper an elementary formal logic of the deontic modalities will be outlined.

<sup>1</sup> For the term "deontic" I am indebted to Professor C. D. Broad.

2. First a preliminary question must be settled. What are the "things" which are pronounced obligatory, permitted, forbidden, etc. ?

We shall call these "things" acts.

The word "act", however, is used ambiguously in ordinary language. It is sometimes used for what might be called act-qualifying properties, *e.g.* theft. But it is also used for the individual cases which fall under these properties, *e.g.* the individual thefts.

The use of the word for individual cases is perhaps more appropriate than its use for properties. For the sake of verbal convenience, however, we shall in this paper use "act" for properties and not for individuals. We shall say that theft, murder, smoking, etc. are acts. The individual cases that fall under theft, murder, smoking, etc. we shall call act-individuals. It is of acts and not of act-individuals that deontic words are predicated.

The performance or non-performance of a certain act (by an agent) we shall call performance-values (for that agent). An act will be called a performance-function of certain other acts, if its performance-value for any given agent uniquely depends upon the performance-values of those other acts for the same agent.

The concept of a performance-function is strictly analogous to the concept of a truth-function in propositional logic.

Particular performance-functions can be defined in strict correspondence to the particular truth-functions.

Thus by the negation (-act) of a given act we understand that act which is performed by an agent, if and only if he does not perform the given act. For example: the negation of the act of repaying a loan is the act of not repaying it. If  $A$  denotes (is the name of) an act,  $\sim A$  will be used as a name of its negation (-act).

Similarly, we can define the conjunction-, disjunction-, implication-, and equivalence-act of two given acts. (The implication-act, *e.g.*, of two given acts is the act which is performed by an agent, if and only if it is not the case that the first act is performed and the second act is not performed by the agent in question.) If  $A$  and  $B$  denote acts,  $A \ \& \ B$  will be used as a name of their conjunction,  $A \ \vee \ B$  as a name of their disjunction,  $A \rightarrow B$  as a name of their implication, and  $A \leftrightarrow B$  as a name of their equivalence.

Finally, we can define the tautology- and contradiction (-act) of  $n$  given acts. The first is the act which is performed and the second the act which is not performed by an agent, whatever be

the performance-values of the  $n$  given acts for the agent in question.

We shall call  $\sim A$  the negation-name of  $A$ , and  $A \& B$  the conjunction-,  $A \vee B$  the disjunction-,  $A \rightarrow B$  the implication-, and  $A \leftrightarrow B$  the equivalence-name of  $A$  and  $B$ .

A name of an act which is neither the negation-name of another name of an act, nor the conjunction-, disjunction-, implication-, or equivalence-name of two other names of acts we shall call an atomic name.

By a molecular complex of  $n$  names of acts we understand :

(i) Any one of the  $n$  names themselves and any one of their negation-names.

(ii) The conjunction-, disjunction-, implication-, and equivalence-name of any two of the  $n$  names.

(iii) The negation-name of any molecular complex of the  $n$  names, and the conjunction-, disjunction-, implication-, and equivalence-name of any two molecular complexes of the  $n$  names.

The  $n$  names are called constituents of their molecular complexes. If they are atomic names, they are called atomic constituents.

As to the use of brackets we adopt the convention that the symbol  $\&$  has a stronger combining force than  $\vee$ ,  $\rightarrow$ , and  $\leftrightarrow$ ; the symbol  $\vee$  than  $\rightarrow$  and  $\leftrightarrow$ ; and the symbol  $\rightarrow$  than  $\leftrightarrow$ . Thus, e.g., we write for  $((A \& B) \vee C) \rightarrow D) \leftrightarrow E$  simply  $A \& B \vee C \rightarrow D \leftrightarrow E$ .

The symbols  $\sim$ ,  $\&$ ,  $\vee$ ,  $\rightarrow$ , and  $\leftrightarrow$  will be used for truth-functions as well as for performance-functions. This ambiguity does not easily lead to confusion and is, therefore, to be preferred to the introduction of two special sets of symbols.

3. As an undefined deontic category we introduce the concept of permission. It is the only undefined deontic category which we need.

If an act is not permitted, it is called forbidden. For instance : Theft is not permitted, hence it is forbidden. We are *not allowed* to steal, hence we *must not* steal.<sup>1</sup>

If the negation of an act is forbidden, the act itself is called obligatory. For instance : it is forbidden to disobey the law, hence it is obligatory to obey the law. We *ought to do* that which we are *not allowed not to do*.

If an act and its negation are both permitted, the act is called

<sup>1</sup> It need hardly be stressed that the question of validity of various deontic propositions (other than those which are true on formal grounds) does not concern us in this paper.

(morally) indifferent. For instance : in a smoking compartment we may smoke, but we may also not smoke. Hence smoking is here a morally indifferent form of behaviour.

It should be observed that indifference is thus a narrower category than permission. Everything indifferent is permitted, but everything permitted is not indifferent. For, what is obligatory is also permitted, but not indifferent.

(The difference between the permitted and the indifferent among the deontic modes is analogous to the difference between the possible and the contingent among the alethic modes.)

The above deontic concepts apply to a single act (or performance-function of acts). There are also deontic concepts which apply to pairs of acts.

Two acts are morally incompatible, if their conjunction is forbidden (and compatible if it is permitted). For instance : giving a promise and not keeping it are (*morally*) incompatible acts.

Doing one act commits us to do another act, if the implication of the two acts is obligatory. For instance : giving a promise commits us to keep it.

The proposition that the act named by  $A$  is permitted will be expressed in symbols by  $PA$ .

The proposition that the act named by  $A$  is forbidden, is the negation of the proposition that it is permitted. It can thus be symbolized by  $\sim(PA)$ .

The proposition that the act named by  $A$  is obligatory, is the negation of the proposition that the negation of the act is permitted. It can thus be symbolized by  $\sim(P\sim A)$ . We shall also use the shorter expression  $OA$ .

The proposition that the act named by  $A$  is (morally) indifferent can be symbolized by  $(PA) \& (P\sim A)$ .

The proposition that the acts named by  $A$  and by  $B$  are (morally) incompatible can be symbolized by  $\sim(PA \& B)$ .

The proposition that the performance of the act named by  $A$  commits us to perform the act named by  $B$  can be symbolized by  $OA \rightarrow B$ . But  $OA \rightarrow B$  means the same as  $\sim(P\sim(A \rightarrow B))$ , and this means the same as  $\sim(PA \& \sim B)$ . Commitment can thus be explained in terms of compatibility.

$P$  and  $O$  are called the deontic operators. Sentences of the type " $P$  ", where a name of an act (or a molecular complex of names of acts) has to be inserted in the blank, we shall call  $P$ -sentences. Similarly, we shall call sentences of the type " $O$  "  $O$ -sentences.

As to the use of brackets it should be remarked that  $P$ - and

O-sentences as constituents of molecular complexes of sentences should be enclosed within brackets in order to avoid confusion. It should further be observed that a deontic operator before a molecular complex of names of acts refers to the whole complex and not to its first constituent only. Thus, e.g.,  $PA \vee B$  means that the act named by  $A \vee B$  is permitted.

The system of Deontic Logic, which we are outlining in this paper, studies propositions (and truth-functions of propositions) about the obligatory, permitted, forbidden, and other (derivative) deontic characters of acts (and performance-functions of acts).

We shall call the propositions which are the object of study deontic propositions. The sentences, in which they are expressed in our system, are *P*- and *O*-sentences or molecular complexes of such sentences.

4. A task of particular importance which Deontic Logic sets itself is to develop a technique for deciding, whether the propositions it studies are logically true or not. (The decision problem.)

Sometimes molecular complexes of *P*- and *O*-sentences express truths of logic for reasons which have nothing to do with the specific character of deontic concepts. For instance: If *A* is permitted, if *B* is permitted, then *B* is forbidden, if *A* is forbidden. In symbols:  $((PB) \rightarrow (PA)) \rightarrow (\sim(PA) \rightarrow \sim(PB))$ . This is a truth of logic. It is an application of a variant of the so called *modus tollens* which is valid for any sentences, whether deontic or not. It is, therefore, a trivial truth from the point of view of our Deontic Logic.

Sometimes, however, molecular complexes of *P*- and *O*-sentences express truths of logic for reasons which depend upon the specific (logical) character of deontic concepts. For instance: If *A* is obligatory and if doing *A* commits us to do *B*, then *B* is obligatory too. In symbols:  $(OA) \& (OA \rightarrow B) \rightarrow (OB)$ . It is intuitively obvious that this is a truth of logic, i.e. something which is valid on purely formal grounds. It is, however, not an application of any scheme which is valid for *any* sentences, whether deontic or not. The existence of logical truths which are peculiar to deontic concepts is what makes the study of Deontic Logic interesting.

If a molecular complex of *P*- and *O*-sentences expresses logical truth for reasons which are independent of the specific nature of deontic concepts, then its truth can be established or proved in a truth-table of propositional logic.

If, however, a molecular complex of *P*- and *O*-sentences

expresses logical truth for reasons which depend on the specific nature of deontic concepts, then its truth cannot be established by the means of propositional logic alone. The question therefore arises: What is the necessary and sufficient criterion which a molecular complex of *P*- and/or *O*-sentences must satisfy in order to express a logically true proposition?

5. Let us call "permitted" and "forbidden" the two deontic values.

An act will be called a deontic function of certain other acts, if the deontic value of the former uniquely depends upon the deontic values of the latter.

It is easy to see that not any act which is a performance-function of certain other acts is also a deontic function of them. (Otherwise the logic of deontic concepts would be trivial.)

Consider first the negation of a given act. From the fact that *A* is performed, we can conclude to the fact that  $\sim A$  is not performed. But from the fact that *A* is permitted, we can conclude nothing as to the permitted or forbidden character of  $\sim A$ . Sometimes  $\sim A$  is permitted, sometimes not. If *A* is what we have called indifferent, then  $\sim A$  is also permitted, but if *A* happened to be obligatory as well as permitted, then  $\sim A$  would be forbidden. In the smoking compartment, *e.g.*, not-smoking is permitted and also smoking. But in the non-smoking compartment, not-smoking is permitted and smoking forbidden.

Consider next the conjunction of two acts. From the fact that *A* and *B* are both performed, it follows that *A* & *B* is performed. But from the fact that *A* and *B* are both permitted, it does not follow that *A* & *B* is permitted. Sometimes *A* & *B* is permitted, sometimes not. For, *A* and *B* may both be permitted, but doing either of them may commit us not to do the other. I may be free to promise and also not to promise to give a certain thing to a person, and free to give and also not to give this thing to him, but forbidden to promise to give and yet not give it.

Consider, finally; the disjunction of two acts. From the fact that at least one of the two acts *A* and *B* is performed, it follows that  $A \vee B$  is performed, and from the fact that none of the two acts *A* and *B* is performed, it follows that  $A \vee B$  is not performed. Similarly, from the fact that at least one of the acts is permitted, it follows that their disjunction is permitted, and from the fact that both acts are forbidden, it follows that their disjunction is forbidden. In other words: the disjunction of two acts is permitted, if and only if at least one of the acts is permitted. Speaking loud or smoking is permitted in the reading-room, if

and only if speaking loud is permitted or smoking is permitted.<sup>1</sup>

Thus deontic functions are similar to performance-functions (and truth-functions) in regard to disjunction, but not similar in regard to negation and conjunction. The similarity can be laid down as a Principle of Deontic Distribution :

*If an act is the disjunction of two other acts, then the proposition that the disjunction is permitted is the disjunction of the proposition that the first act is permitted and the proposition that the second act is permitted.*

(This principle can, naturally, be extended to disjunctions with any number  $n$  of members.)

In virtue of familiar principles of formal logic, any molecular complex of  $n$  names of acts has what we propose to call a perfect disjunctive normal form. This is a 0-, 1-, or more-than-1-termed disjunction-name of  $n$ -termed conjunction-names. Each of the  $n$  original names or its negation-name occurs in every one of the conjunction-names.

In virtue of the above Principle of Deontic Distribution, any molecular complex of  $n$  names of acts denotes a deontic function of the acts denoted by the conjunction-names in its perfect disjunctive normal form.

Consider now a  $P$ -sentence  $Pc$ , where  $c$  stands for (an atomic name of an act or) a molecular complex of names of acts. Let  $c_1, \dots, c_k$  stand for the conjunction-names in the perfect disjunctive normal form of  $c$ . The sentences  $Pc_1, \dots, Pc_k$  we shall call the  $P$ -constituents of  $Pc$ .

Since, in virtue of the Principle of Deontic Distribution,  $c$  denotes a deontic function of the acts named by  $c_1, \dots, c_k$ , it follows that  $Pc$  expresses a truth-function of the propositions expressed by  $Pc_1, \dots, Pc_k$ . Generally speaking: a  $P$ -sentence expresses a truth-function of the propositions expressed by its  $P$ -constituents.

Consider  $n$  names of acts  $A_1, \dots, A_n$ . There are in all  $2^n$  conjunction-names which can be formed by selecting  $m$  ( $0 \leq m \leq n$ ) out of the  $n$  names and taking the negation-names of the

<sup>1</sup> The meaning of "or" in ordinary language is not quite settled. When we say that we are permitted to do  $A$  or  $B$ , we sometimes mean, by implication, that we are allowed to do both. Sometimes, however, we mean that we are allowed to do one and one only of the two acts. Which meaning the "or" conveys by implication depends upon the material nature of the individual case, in which it is used. It ought to be stressed that our use of "or" in this paper is neutral with regard to such material differences in the individual situations. That we are permitted to do  $A$  or  $B$  means here that we are permitted to do at least one of the two acts, and neither excludes nor includes, by implication, the permission to do both.

remaining  $n-m$  names. (The order of the names in a conjunction-name is irrelevant.) By the deontic units in the deontic realm of the acts named by  $A_1, \dots, A_n$  we shall understand the propositions that the respective acts named by those  $2^n$  conjunction-names are permitted. By the deontic realm itself we shall understand the disjunction of all the deontic units.

Thus, *e.g.*, the deontic units of the deontic realm of the sole act named by  $A$  are the propositions expressed by  $PA$  and  $P\sim A$ . The deontic realm itself is the proposition expressed by  $(PA) \vee (P\sim A)$ . The deontic units of the deontic realm of the acts named by  $A$  and  $B$  are the propositions expressed by  $PA \& B$  and  $PA \& \sim B$  and  $P\sim A \& B$  and  $P\sim A \& \sim B$ . Etc.

The deontic units of the deontic realm of given acts are logically independent of one another, meaning that they can be true or false in any combination of truth-values. There is, however, one point at which this independence might be questioned. Could *all* the deontic units be false?

Let  $A$  be the name of an act. That all (both) the deontic units in the deontic realm of this act are false means that the act itself and its negation are both forbidden. In symbols:  $\sim(PA) \& \sim(P\sim A)$ . Since the act *or* its negation is performed by any agent whenever he acts, the falsehood of all the deontic units means that we are forbidden to act in any way whatsoever.

Is such a prohibition illogical? Its counterpart in the logic of the alethic modalities would be the case, when a proposition and its negation are both impossible, and its counterpart in the logic of the epistemic modalities would be the case, when a proposition and its negation are both known to be false. *These* cases are obviously logical impossibilities. On the other hand, in the logic of the existential modalities the corresponding case is, when a property and its negation are both empty. *This* is not an impossibility, since the Universe of Discourse may have no members. The question, therefore is, whether the deontic modes at this point resemble the alethic and the epistemic modes or whether they resemble the existential modes.

Ordinary language and our common sense logical intuitions seem at first not to provide us with a clear answer. A simple logical transformation, however, will help us to make up our minds.

That the negation of an act is forbidden means that the act itself is obligatory. Thus we can for  $\sim(P\sim A)$  write  $OA$ . That an act and its negation are both forbidden means the same as that the act itself is both obligatory and forbidden.

At this point an appeal to ordinary language will, I think, be decisive. We seem prepared to reject a use of the words, according to which one and the same act could be truly called both obligatory and forbidden.<sup>1</sup> If, however, we reject this use, we must also reject the idea that all the units in a deontic realm could be false.

Thus, on the point at issue, the deontic modalities appear to resemble the alethic and the epistemic modalities rather than the existential ones.

The restriction on the logical independence of the deontic units, which we are forced to accept, can be laid down as a Principle of Permission :

*Any given act is either itself permitted or its negation is permitted.*

There are alternative formulations of the principle. We might also have said : If the negation of an act is forbidden, then the act itself is permitted. And this again is equivalent to saying : If an act is obligatory, then it is permitted.

6. Which truth-function of its *P*-constituents a *P*-sentence expresses can be investigated and decided in truth-tables.

We shall here construct a truth-table for the following *P*-sentences :  $PA$  and  $P\sim A$  and  $PA \& B$  and  $PA \vee B$  and  $PA \rightarrow B$  and  $PA \leftrightarrow B$  and  $PA \vee \sim A$ . The perfect disjunctive normal form of  $A$  (in terms of  $A$  and  $B$ ) is  $A \& B \vee A \& \sim B$ . The normal form of  $\sim A$  is  $\sim A \& B \vee \sim A \& \sim B$ .

The table looks as follows :

$PA \& B$	$PA \& \sim B$	$P\sim A \& B$	$P\sim A \& \sim B$	$PA$	$P\sim A$	$PA \& B$	$PA \vee B$	$PA \rightarrow B$	$PA \leftrightarrow B$	$PA \vee \sim A$
T	T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T	T	T	T
T	T	F	F	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T	T	T	T
T	F	T	F	T	T	T	T	T	T	T
T	F	F	T	T	T	T	T	T	T	T
T	F	F	F	T	F	T	T	T	T	T
F	T	T	T	T	T	F	T	T	T	T
F	T	T	F	T	T	F	T	T	F	T
F	T	F	T	T	T	F	T	T	T	T
F	T	F	F	T	F	F	T	F	F	T
F	F	T	T	F	T	F	T	T	T	T
F	F	T	F	F	T	F	T	T	F	T
F	F	F	T	F	T	F	F	T	T	T
F	F	F	F	F	T	F	F	T	T	T

<sup>1</sup> For the "relativity" of deontic propositions cf. below, p. 15.

The normal form of  $A \& B$  is  $A \& B$ . The normal form of  $A \vee B$  is  $A \& B \vee A \& \sim B \vee \sim A \& B$ . The normal form of  $A \rightarrow B$  is  $A \& B \vee A \& \sim B \vee \sim A \& \sim B$ . The normal form of  $A \leftrightarrow B$  is  $A \& B \vee \sim A \& \sim B$ . The normal form of  $A \vee \sim A$  is  $A \& B \vee A \& \sim B \vee \sim A \& B \vee \sim A \& \sim B$ . Thus the seven  $P$ -sentences have in all four  $P$ -constituents, *viz.*  $PA \& B$  and  $PA \& \sim B$  and  $P \sim A \& B$  and  $P \sim A \& \sim B$ . They express the deontic units of the deontic realm of the two acts named by  $A$  and by  $B$ .

In distributing truth-values over the deontic units (or the  $P$ -constituents) we have to observe the restriction imposed by the Principle of Permission. The subsequent calculation of truth-values for the seven deontic propositions (or the seven  $P$ -sentences) depends only on the Principle of Deontic Distribution.

What is the truth-table for  $PA \& \sim A$ ? The perfect disjunctive normal form of  $A \& \sim A$  is "empty", *i.e.* a  $O$ -termed disjunction. Thus  $PA \& \sim A$  too is a  $O$ -termed disjunction of  $P$ -constituents. It might be argued that a disjunction is true, if and only if at least one of its members is true, and that a  $O$ -termed disjunction, since it has no members, is never true (always false). If, however,  $PA \& \sim A$  is always false, its negation  $\sim(PA \& \sim A)$  is always true. But  $\sim(PA \& \sim A)$  means the same as  $OA \vee \sim A$ . Thus, on the above criterion for the truth of a  $O$ -termed disjunction, it follows that  $OA \vee \sim A$  is a deontic tautology.

It might, however, be questioned whether it can be regarded as a truth of logic that a tautologous act is obligatory (and a contradictory act forbidden). The corresponding proposition in the logic of the alethic modalities is that a tautologous proposition is necessary (and a contradictory proposition impossible), and the corresponding proposition in the logic of the existential modalities is that a tautologous property is universal (and a contradictory property empty). *These* corresponding cases are obviously logical truths. On the other hand, the corresponding propositions in the logic of the epistemic modalities is that a tautologous proposition is verified (and a contradictory proposition falsified). *This* is not a logical truth. For, a proposition may be tautologous (contradictory) without our knowing it. The question therefore is, whether the deontic modes at this point resemble the alethic and the existential modes, or whether they resemble the epistemic modes.

Ordinary language and our common sense logical intuitions seem not to provide us with any clear answer. It appears, moreover, that no further logical considerations can help us to

decide on the issue. It may be thought "awkward" to permit contradictory actions<sup>1</sup> but it is difficult to conceive of any logical argument against this permission. From the point of view of logic, therefore, the most plausible course seems to be to regard  $PA \ \& \ \sim A$  and  $OA \ \vee \ \sim A$  as expressing contingent propositions which can be either true or false.

Thus, on the point at issue, the deontic modalities appear to resemble the epistemic rather than the alethic and the existential modalities.

We suggest the following Principle of Deontic Contingency :

*A tautologous act is not necessarily obligatory, and a contradictory act is not necessarily forbidden.*

7. Let us consider a molecular complex of  $P$ - and/or  $O$ -sentences.  $O$ -sentences can be regarded as abbreviations for negation-sentences of certain  $P$ -sentences. (Cf. above, p. 8.) If the molecular complex happened to contain  $O$ -sentences, we replace them by negation-sentences of  $P$ -sentences. Thus we get a new molecular complex, all the constituents of which are  $P$ -sentences.

We now turn our attention to the (molecular complexes of) names of acts which follow after the modal operators in this new molecular complex of  $P$ -sentences. We make an inclusive list of all atomic names which are constituents of at least one of the (molecular complexes of) names of acts in question. Thereupon we transform these (molecular complexes of) names of acts into their perfect disjunctive normal forms in terms of all atomic names which occur in our list. The respective conjunction-names in these normal forms preceded by the deontic operator  $P$  we shall call the  $P$ -constituents of the initially given molecular complex of  $P$ - and/or  $O$ -sentences. (Cf. the example given below).

We know already that any  $P$ -sentence expresses a truth-function of the propositions expressed by its  $P$ -constituents. Since any molecular complex of  $P$ - and/or  $O$ -sentences expresses a truth-function of the propositions expressed by the  $P$ - and/or  $O$ -sentences themselves, it follows that any molecular complex of  $P$ - and/or  $O$ -sentences expresses a truth-function of the propositions expressed by its  $P$ -constituents.

Which truth-function of the propositions expressed by its  $P$ -constituents a molecular complex of  $P$ - and/or  $O$ -sentences expresses can be investigated and decided in a truth-table.

<sup>1</sup> Contradictory acts should not be confused with (morally) indifferent acts. The former are acts which, by definition, are never performed by an agent. The latter are acts which we are permitted to perform, but also not to perform.

This fact constitutes a solution of the decision problem for the system of Deontic Logic which we are outlining in this paper.

The technique of constructing truth-tables in Deontic Logic will be illustrated by an example.

Let the molecular complex be  $(O A) \& (O A \rightarrow B) \rightarrow (O B)$ . (Cf. above, p. 5.)

$O A$  is an abbreviation for  $\sim(P \sim A)$ ,  $O A \rightarrow B$  for  $\sim(P A \& \sim B)$ , and  $O B$  for  $\sim(P \sim B)$ . By replacing  $O$ -sentences by  $P$ -sentences in our initial complex, we get the new complex  $\sim(P \sim A) \& \sim(P A \& \sim B) \rightarrow \sim(P \sim B)$ .

The atomic names of acts which are constituents of (at least one of) the molecular complexes "inside" the operator  $P$  are  $A$  and  $B$ . The perfect disjunctive normal form of  $\sim A$  in terms of  $A$  and  $B$  is  $\sim A \& B \vee \sim A \& \sim B$ . The normal form of  $A \& \sim B$  is  $A \& \sim B$ . The normal form of  $\sim B$  is  $A \& \sim B \vee \sim A \& \sim B$ . The  $P$ -constituents of the initially given molecular complex, therefore, are  $P \sim A \& B$  and  $P A \& \sim B$  and  $P \sim A \& \sim B$ .

Since the  $P$ -constituents do not represent all the deontic units of the deontic realm of the acts named by  $A$  and by  $B$  (cf. above p. 10) the Principle of Permission does not here impose any restrictions upon the combinations of truth-values. The calculation of truth-values depends only upon the Principle of Deontic Distribution (and principles of propositional logic). The table looks as follows:

$P A \& \sim B$	$P \sim A \& B$	$P \sim A \& \sim B$	$O A$	$O A \rightarrow B$	$(O A) \& (O A \rightarrow B)$	$O B$	$\rightarrow$
T	T	T	F	F	F	F	T
T	T	F	F	F	F	F	T
T	F	T	F	F	F	F	T
T	F	F	T	F	F	F	T
F	T	T	F	T	F	F	T
F	T	F	F	T	F	T	T
F	F	T	F	T	F	F	T
F	F	F	T	T	T	T	T

It is seen that the molecular complex which we are investigating (indicated by " $\rightarrow$ " in the column to the extreme right) expresses the tautology of the propositions expressed by its  $P$ -constituents.

8. A molecular complex of  $P$ - and/or  $O$ -sentences which expresses the tautology of the propositions expressed by its  $P$ -constituents, is said to express a truth of Deontic Logic or a deontic tautology.

A (true) proposition to the effect that a certain molecular complex of *P*- and/or *O*-sentences expresses a deontic tautology will be called a law of Deontic Logic.

We mention below some examples of such laws. When we call two molecular complexes of *P*- and/or *O*-sentences identical, we mean that their equivalence-sentence expresses a deontic tautology. When we say that (the proposition expressed by) one molecular complex of *P*- and/or *O*-sentences entails (the proposition expressed by) another, we mean that their implication-sentence expresses a deontic tautology. The propositions expressed by the molecular complexes of sentences given below (or by the equivalence- or implication-sentences in question) are easily shown by truth-tables to be tautologies.

(i) Two laws on the relation of permission to obligation, and *vice versa* :

*a*  $PA$  is identical with  $\sim(O \sim A)$ , i.e.  $(PA) \leftrightarrow \sim(O \sim A)$  expresses a deontic tautology.

*b*  $OA$  entails  $PA$ , i.e.  $(OA) \rightarrow (PA)$  expresses a deontic tautology.

The second of these laws should not be confused with the above (alternative formulation of the) Principle of Permission (p. 9). In the proof of (i)*b* this principle is already assumed.

(ii) Four laws for the "dissolution" of deontic operators :

*a*  $OA \& B$  is identical with  $(OA) \& (OB)$ .

*b*  $PA \vee B$  is identical with  $(PA) \vee (PB)$ .

*c*  $(OA) \vee (OB)$  entails  $OA \vee B$ .

*d*  $PA \& B$  entails  $(PA) \& (PB)$ .

The second of these laws should not be confused with the Principle of Deontic Distribution (p. 7). In the proof of (ii)*b* this principle is already assumed.

(iii) Six laws on "commitment" :

*a*  $(OA) \& (OA \rightarrow B)$  entails  $OB$ . If doing what we ought to do commits us to do something else, then this new act is also something which we ought to do. (This was the example of a deontic tautology which we discussed above.)

*b*  $(PA) \& (OA \rightarrow B)$  entails  $PB$ . If doing what we are free to do commits us to do something else, then this new act is also something which we are free to do. In other words : doing the permitted can never commit us to do the forbidden.

*c*  $\sim(PB) \& (OA \rightarrow B)$  entails  $\sim(PA)$ . This is but a new version of the previous law. If doing something commits us to do the forbidden, then we are forbidden to do the first thing. For instance : if it is obligatory to keep one's promises and if we

promise to do something which is forbidden, then the act of promising this thing is itself forbidden.

$d (O A \rightarrow B \vee C) \& \sim (P B) \& \sim (P C)$  entails  $\sim (P A)$ . This is a further version of the two previous laws. An act which commits us to a choice between forbidden alternatives is forbidden.

$e \sim ((O A \vee B) \& \sim (P A) \& \sim (P B))$ . It is logically impossible to be obliged to choose between forbidden alternatives.<sup>1</sup>

$f (O A) \& (O A \& B \rightarrow C)$  entails  $O B \rightarrow C$ . If doing two things, the first of which we ought to do, commits us to do a third thing, then doing the second thing alone commits us to do the third thing. "Our commitments are not affected by our (other) obligations."

$g O \sim A \rightarrow A$  entails  $O A$ . If failure to perform an act commits us to perform it, then this act is obligatory.

The truth of all these laws follows from our intuitive notions of obligation and permission. Not all of the laws themselves, however, are intuitively obvious. In the case of some of the laws, moreover, it is not intuitively clear whether their truth is a matter of logic or a matter of moral code. This proves that the decision procedure of Deontic Logic which we have outlined is not void of philosophical interest.

Any molecular complex of  $P$ - and/or  $O$ -sentences has what we propose to call an absolutely perfect disjunctive normal form. This we get by replacing every one of the  $P$ - and/or  $O$ -sentences by a disjunction-sentence of  $P$ -constituents of the complex and transforming the molecular complex of  $P$ -sentences thus obtained into its perfect disjunctive normal form. If the normal form contains the conjunction of the negation of all  $P$ -constituents, we omit it from the normal form.

The absolutely perfect disjunctive normal form shows with which ones of the possible combinations of truth-values in its  $P$ -constituents the molecular complex in question expresses agreement and with which ones it expresses disagreement. If it agrees with all possibilities, it expresses a deontic tautology, *i.e.* is a truth of Deontic Logic.

<sup>1</sup> Aquinas several times refers to the laws  $d$  and  $e$ . He distinguishes between a man's being *perplexus simpliciter* and his being *perplexus secundum quid*. The former is the case, if he is, as such, obliged to choose between forbidden alternatives. The latter is the case, if he by a previous wrong act commits himself to a choice between forbidden alternatives. Aquinas rightly denies that a man could be *perplexus simpliciter* ( $e$ ) and affirms that a man might be *perplexus secundum quid* ( $d$ ). Cf. *De Veritate*, Q. 17, art. 4; *Summa Theologica*, jajar, Q. 19, art. 6; *Summa Theologica*, iii<sup>a</sup>, Q. 64, art. 6. For these observations I am indebted to Mr. P. Geach.

9. There is one relevant respect, in which the deontic modalities differ from the alethic, epistemic, and existential modalities. It can be illustrated as follows: If a proposition is true, then it is possible, and if a proposition is true, then it is not falsified, and if a property is true of a thing, then the property exists. But if an act is performed (or not performed), then nothing follows as regards its obligatory, permitted or forbidden character. There is thus an important sense in which the deontic modalities unlike the alethic, epistemic, and existential ones have no logical connexions with matters of fact (truth and falsehood). This is a point about deontic categories which has often been stressed by moral philosophers.

10. In this paper deontic propositions have been treated as "absolute". They can, however, be made "relative" in several ways.

First of all, it might be argued that deontic propositions are sometimes, or perhaps always, relative to some so-called moral code. What is obligatory within one moral code, may be forbidden within another.

Secondly, instead of simply considering whether an act is obligatory, permitted or forbidden, we may consider propositions of the following type:  $x$  is permitted to do  $A$ , or  $x$  permits  $y$  to do  $A$ . Introducing quantifiers we then get propositions of the type: somebody is permitted to do  $A$ , or somebody permits everybody to do  $A$ , etc. The logical systems which we get by such extensions are of considerable complexity. Their decision-problem can be solved for many interesting cases, but not for all cases.

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